Methods of D al Filtering and Multi-Dimensional Data ompression
Using the Farey Quadrature and Arithmetic, Fan, and Modular Wavelets

Claims

I claim:

1. A method of compressing and reconstructing input data comprising the steps of: providing input data in an initial form; sampling the input data at the points of the Farey quadrature; transforming the Farey-sampled data into an intermediate form; quantizing the Farey-sampled data while in the intermediate form; using the quantized intermediate form to store or transmit the Farey-sampled data; de-quantizing the quantized intermediate form of the Farey-sampled data prior to reconstructing the data; and reconstructing the intermediate form of the Farey-sampled data in order to obtain reconstructed data for use.

2. A method as recited in claim 1 wherein: the step of transforming the Farey-sampled data into an intermediate form is achieved via a wavelet transformation; and the intermediate form of the Farey-sampled data takes the form of wavelet coefficients.

3. A method as recited in claim 2 wherein the wavelet transformation is based on arithmetic wavelets.

4. A method as recited in claim 3 wherein the arithmetic wavelets are defined in accordance with the following expressions:

$$\tilde{\vartheta}_I(\theta) = \begin{cases} +2\cos\theta + 2\sin\theta - 2; & \text{if } 0 \le \theta \le \frac{\pi}{2}, \\ +2\cos\theta - 2\sin\theta + 2; & \text{if } \frac{\pi}{2} \le \theta \le \pi, \\ -2\cos\theta - 2\sin\theta - 2; & \text{if } \pi \le \theta \le \frac{3\pi}{2}, \\ -2\cos\theta + 2\sin\theta + 2; & \text{if } \frac{3\pi}{2} \le \theta \le 2\pi; \end{cases}$$

and if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ labels an arithmetic arrow;

and if A labels a top arithmetic arrow and $a > c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(d^2 - c^2)\cos\theta + 8cd\sin\theta - 4(d^2 + c^2); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2 - c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2 - (a+c)^2}, \\ +2(a^2 + d^2 - b^2 - c^2 + 2ac - 2bd)\cos\theta \\ \quad +4(cd - ab - bc - ad)\sin\theta \\ \quad + 2(a^2 + b^2 - c^2 - d^2 + 2ac + 2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2 - (a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2 - a^2}, \\ +2(b^2 + d^2 - a^2 - c^2 + 2ac - 2bd)\cos\theta \\ \quad +4(ab + cd - ad - bc)\sin\theta \\ \quad + 2(-a^2 - b^2 - c^2 - d^2 + 2ac + 2bd); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2 - a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2 - (c-a)^2}, \\ 0; \qquad \text{otherwise}; \end{cases}$$

and if A labels a top arithmetic arrow and $c \geq a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2 + c^2 - b^2 - d^2 - 2ac + 2bd) \cos \theta \\ \quad +4(ad + bc - ab - cd) \sin \theta \\ \quad +2(a^2 + b^2 + c^2 + d^2 - 2ac - 2bd); \\ \qquad\qquad \text{if } \tan^{-1} \frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \leq \theta \leq \tan^{-1} \frac{2cd}{d^2-c^2}, \\ +2(a^2 + d^2 - b^2 - c^2 + 2bd - 2ac) \cos \theta \\ \quad +4(ad + bc + cd - ab) \sin \theta \\ \quad + 2(a^2 + b^2 - c^2 - d^2 - 2bd - 2ac); \\ \qquad\qquad \text{if } \tan^{-1} \frac{2cd}{d^2-c^2} \leq \theta \leq \tan^{-1} \frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +4(a^2 - b^2) \cos \theta \ - \ 8ab \sin \theta \ + \ 4(a^2 + b^2); \\ \qquad\qquad \text{if } \tan^{-1} \frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \leq \theta \leq \tan^{-1} \frac{2ba}{b^2-a^2}, \\ 0; \qquad\qquad \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $a > -c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2 + c^2 - b^2 - d^2 + 2ac - 2bd) \cos \theta \\ \quad +4(-ad - bc - ab - cd) \sin \theta \\ \quad +2(a^2 + b^2 + c^2 + d^2 + 2ac + 2bd); \\ \qquad\qquad \text{if } \tan^{-1} \frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \leq \theta \leq \tan^{-1} \frac{2ba}{b^2-a^2}, \\ +2(b^2 + c^2 - a^2 - d^2 + 2ac - 2bd) \cos \theta \\ \quad +4(ab - ad - bc - cd) \sin \theta \\ \quad + 2(c^2 + d^2 - a^2 - b^2 + 2ac + 2bd); \\ \qquad\qquad \text{if } \tan^{-1} \frac{2ba}{b^2-a^2} \leq \theta \leq \tan^{-1} \frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +4\ (c^2 - d^2) \cos \theta \ - \ 8cd \sin \theta \ + \ 4(c^2 + d^2); \\ \qquad\qquad \text{if } \tan^{-1} \frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \leq \theta \leq \tan^{-1} \frac{2cd}{d^2-c^2}, \\ 0; \qquad\qquad \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $-c \geq a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(b^2 - a^2) \cos \theta \ + \ 8ab \sin \theta \ - \ 4(a^2 + b^2); \\ \qquad\qquad \text{if } \tan^{-1} \frac{2ba}{b^2-a^2} \leq \theta \leq \tan^{-1} \frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +2(b^2 + c^2 - a^2 - d^2 + 2bd - 2ac) \cos \theta \\ \quad +4(ad + bc + ab - cd) \sin \theta \\ \quad + 2(c^2 + d^2 - a^2 - b^2 - 2ac - 2bd); \\ \qquad\qquad \text{if } \tan^{-1} \frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \leq \theta \leq \tan^{-1} \frac{2cd}{d^2-c^2}, \\ +2(b^2 + d^2 - a^2 - c^2 + 2bd - 2ac) \cos \theta \\ \quad +4(ad + bc + cd + ab) \sin \theta \\ \quad + 2(-a^2 - b^2 - c^2 - d^2 - 2ac - 2bd); \\ \qquad\qquad \text{if } \tan^{-1} \frac{2cd}{d^2-c^2} \leq \theta \leq \tan^{-1} \frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}. \\ 0; \qquad\qquad \text{otherwise.} \end{cases}$$

5. A method as reci in claim 2 wherein the wavelet transfc .ation is based on modular wavelets.

6. A method as recited in claim 5 wherein the modular wavelets are defined in accordance with the following expressions:

$$\psi_A(\theta) = \sum_{n\geq 0} n\,\bar{\vartheta}_{U^nA}(\theta),$$
$$\psi_{SA}(\theta) = \sum_{n\geq 0} n\,\bar{\vartheta}_{T^{-n}A}(\theta),$$

where

$$\bar{\vartheta}_A(\theta) = \begin{cases} \tilde{\vartheta}_A(\theta); & \text{if } A \neq U^{-1}, T^{-1}, \\ \tilde{\vartheta}_{U^{-1}}(\theta) - 2\sin\theta; & \text{if } A = U^{-1}, \\ \tilde{\vartheta}_{T^{-1}}(\theta) + 2\sin\theta; & \text{if } A = T^{-1}, \end{cases}$$

with $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $U = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$; and

$$\tilde{\vartheta}_I(\theta) = \begin{cases} +2\cos\theta + 2\sin\theta - 2; & \text{if } 0 \leq \theta \leq \frac{\pi}{2}, \\ +2\cos\theta - 2\sin\theta + 2; & \text{if } \frac{\pi}{2} \leq \theta \leq \pi, \\ -2\cos\theta - 2\sin\theta - 2; & \text{if } \pi \leq \theta \leq \frac{3\pi}{2}, \\ -2\cos\theta + 2\sin\theta + 2; & \text{if } \frac{3\pi}{2} \leq \theta \leq 2\pi; \end{cases}$$

and if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ labels an arithmetic arrow;

and if A labels a top arithmetic arrow and $a > c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(d^2-c^2)\cos\theta + 8cd\sin\theta - 4(d^2+c^2); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \leq \theta \leq \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +2(a^2+d^2-b^2-c^2+2ac-2bd)\cos\theta \\ \quad +4(cd-ab-bc-ad)\sin\theta \\ \quad +2(a^2+b^2-c^2-d^2+2ac+2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \leq \theta \leq \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+d^2-a^2-c^2+2ac-2bd)\cos\theta \\ \quad +4(ab+cd-ad-bc)\sin\theta \\ \quad +2(-a^2-b^2-c^2-d^2+2ac+2bd); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \leq \theta \leq \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ 0; \qquad \text{otherwise}; \end{cases}$$

and if A labels a top arithmetic arrow and $c \geq a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2-2ac+2bd)\cos\theta \\ \quad +4(ad+bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(a^2+d^2-b^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd-ab)\sin\theta \\ \quad +\ 2(a^2+b^2-c^2-d^2-2bd-2ac); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +4(a^2-b^2)\cos\theta \ - \ 8ab\sin\theta \ + \ 4(a^2+b^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ 0; \qquad\qquad \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $a > -c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(-ad-bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+c^2-a^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(ab-ad-bc-cd)\sin\theta \\ \quad +\ 2(c^2+d^2-a^2-b^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +4\ (c^2-d^2)\cos\theta \ - \ 8cd\sin\theta \ + \ 4(c^2+d^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ 0; \qquad\qquad \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $-c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(b^2-a^2)\cos\theta \ + \ 8ab\sin\theta \ - \ 4(a^2+b^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +2(b^2+c^2-a^2-d^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+ab-cd)\sin\theta \\ \quad +\ 2(c^2+d^2-a^2-b^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(b^2+d^2-a^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd+ab)\sin\theta \\ \quad +\ 2(-a^2-b^2-c^2-d^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ 0; \qquad\qquad \text{otherwise.} \end{cases}$$

7. A method as recite · ı claim 2 wherein the wavelet transform. ›n is based on fan wavelets.

8. A method as recited in claim 7 wherein the fan wavelets are defined in accordance with the following expressions:

$$\phi_A(\theta) = \sum_{n\geq 0} \overline{\vartheta}_{U^nA}(\theta),$$
$$\phi_{SA}(\theta) = \sum_{n\geq 0} \overline{\vartheta}_{T^{-n}A}(\theta),$$

where

$$\overline{\vartheta}_A(\theta) = \begin{cases} \tilde{\vartheta}_A(\theta); & \text{if } A \neq U^{-1}, T^{-1}, \\ \tilde{\vartheta}_{U^{-1}}(\theta) - 2\sin\theta; & \text{if } A = U^{-1}, \\ \tilde{\vartheta}_{T^{-1}}(\theta) + 2\sin\theta; & \text{if } A = T^{-1}, \end{cases}$$

with $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $U = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$; and

$$\tilde{\vartheta}_I(\theta) = \begin{cases} +2\cos\theta + 2\sin\theta - 2; & \text{if } 0 \leq \theta \leq \frac{\pi}{2}, \\ +2\cos\theta - 2\sin\theta + 2; & \text{if } \frac{\pi}{2} \leq \theta \leq \pi, \\ -2\cos\theta - 2\sin\theta - 2; & \text{if } \pi \leq \theta \leq \frac{3\pi}{2}, \\ -2\cos\theta + 2\sin\theta + 2; & \text{if } \frac{3\pi}{2} \leq \theta \leq 2\pi; \end{cases}$$

and if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ labels an arithmetic arrow;

and if A labels a top arithmetic arrow and $a > c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(d^2-c^2)\cos\theta + 8cd\sin\theta - 4(d^2+c^2); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \leq \theta \leq \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +2(a^2+d^2-b^2-c^2+2ac-2bd)\cos\theta \\ \quad +4(cd-ab-bc-ad)\sin\theta \\ \quad +2(a^2+b^2-c^2-d^2+2ac+2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \leq \theta \leq \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+d^2-a^2-c^2+2ac-2bd)\cos\theta \\ \quad +4(ab+cd-ad-bc)\sin\theta \\ \quad +2(-a^2-b^2-c^2-d^2+2ac+2bd); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \leq \theta \leq \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ 0; \qquad \text{otherwise;} \end{cases}$$

and if A labels a top arithmetic arrow and $c \geq a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2-2ac+2bd)\cos\theta \\ \quad +4(ad+bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2-2ac-2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(a^2+d^2-b^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd-ab)\sin\theta \\ \quad +2(a^2+b^2-c^2-d^2-2bd-2ac); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +4(a^2-b^2)\cos\theta \; - \; 8ab\sin\theta \; + \; 4(a^2+b^2); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ 0; \qquad \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $a > -c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(-ad-bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2+2ac+2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+c^2-a^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(ab-ad-bc-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2+2ac+2bd); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +4\,(c^2-d^2)\cos\theta \; - \; 8cd\sin\theta \; + \; 4(c^2+d^2); \\ \qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ 0; \qquad \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $-c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(b^2-a^2)\cos\theta \; + \; 8ab\sin\theta \; - \; 4(a^2+b^2); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +2(b^2+c^2-a^2-d^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+ab-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2-2ac-2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(b^2+d^2-a^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd+ab)\sin\theta \\ \quad +2(-a^2-b^2-c^2-d^2-2ac-2bd); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ 0; \qquad \text{otherwise.} \end{cases}$$

9. A method as recited claim 2 wherein the steps of sampling tl .nput data at the points of the Farey quadrature and transforming the Farey-sampled data into wavelet coefficients is characterized by a binary cascade of arrow structures arising from the calculation of wavelet coefficients by sampling the input data in its initial form at the points of the Farey quadrature as defined along a circle in the complex plane.

10. A method as recited in claim 2 wherein the wavelet transformation is based on arithmetic wavelets and the step of transforming the Farey-sampled data into arithmetic wavelet coefficients includes the step of regularization.

11. A method as recited in claim 2 wherein the wavelet transformation and the reconstruction are based on a finite number of wavelets.

12. A method as recited in claim 2 wherein: wavelet transformation is achieved via a plurality of wavelet transformation operations each for a specified arc along a circle in the complex plane; and the plurality of wavelet transformation operations are calculated in parallel with each other.

13. A method as recited in claim 1 wherein the quantized intermediate form of the data is stored and retrieved prior to de-quantizing and reconstructing the data for use.

14. A method as recited in claim 1 wherein: the steps of transforming the Farey-sampled data into the intermediate form and quantizing the intermediate form of the Farey-sampled data occur at a first physical location; the quantized intermediate form of the data is transmitted to a second physical location; and the steps of de-quantizing the quantized intermediate form of the Farey-sampled data and reconstructing the intermediate form of the Farey-sampled data in order to obtain reconstructed data occurs at the second physical location.

15. A method as recited in claim 14 further comprising the step of transmitting a reconstruction algorithm to the second physical location in order to accomplish reconstruction at the second physical location.

16. A method as recited in claim 1 wherein the input data is digital data.

17. A method as recited in claim 1 wherein the input data is analog data and the analog input data is sampled by placing analog sensors at locations corresponding to the points of the Farey quadrature.

18. A method as recited in claim 1 wherein the input data and the reconstructed data are multi-dimensional.

19. A method as recited in claim 2 wherein the step of reconstructing the intermediate form of the Farey-sampled data in order to obtain the reconstructed data is achieved via an inverse wavelet transformation which is characterized by a binary cascade.

20. A method as recited in claim 2 wherein the calculation of the wavelet transformation

includes the step of rei malization.

21. A method as recited in claim 1 wherein the sampling of data at the points of the Farey quadrature includes temporary storage of sequences of data in a buffer.

22. A digital signal processing method for obtaining data in a transform domain comprising the steps of: providing input data in an initial form; sampling the input data at the points of the Farey quadrature; using an intermediate transform to transform the Farey-sampled data into an intermediate form; and using a primary transform to convert the data in intermediate form of the Farey-sampled data into transform coefficients representative of the data in a transform domain.

23. A method as recited in claim 22 wherein the primary transformation consists of one of the groups of following transformations: Fourier, Hilbert, Haar, Laplace, Bessel, Laguerre, Hermite, Chebyshev, Hotelling, Mersenne and Fermat.

24. A method as recited in claim 22 wherein the primary transformation consists of the Fourier transform, and the intermediate transform is a wavelet transformation based on arithmetic wavelets.

25. A method as recited in claim 24 wherein the intermediate transform is an arithmetic wavelet transform and the primary transform is a Fourier transform, and the conversion of the data from the intermediate form to the Fourier coefficients, c_n^A, is given by the following expressions:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \text{ labels an arithmetic arrow; and}$$

$$\begin{aligned} \pi\, i\, (n^3 - n)\ c_n^A \ = \ & -[(c-a)^2 + (b-d)^2]\left[\frac{(b-d) - i(a-c)}{(b-d) + i(a-c)}\right]^n \\ & + 2(c^2+d^2)\left[\frac{d-ic}{d+ic}\right]^n \ + \ 2(a^2+b^2)\left[\frac{b-ia}{b+ia}\right]^n \\ & - [(c+a)^2 + (b+d)^2]\left[\frac{(b+d) - i(a+c)}{(b+d) + i(a+c)}\right]^n; \end{aligned}$$

and the coefficients c_0^A, c_1^A, c_{-1}^A are given by

$$\begin{aligned} \pi c_{\pm 1}^A = {} & \theta_h(c^2 - d^2 \pm 2icd) \ + \ \theta_r[(b-d)^2 - (a-c)^2 \mp 2i(b-d)(a-c)]/2 \\ & + \ \theta_t(a^2 - b^2 \pm 2iab) \ + \ \theta_\ell[(b+d)^2 - (a+c)^2 \mp 2i(b+d)(a+c)]/2, \end{aligned}$$

$$\begin{aligned} \pi c_0^A = {} & 2\theta_h(c^2 + d^2) \ - \ \theta_r[(b-d)^2 + (a-c)^2] \\ & + 2\theta_t(a^2 + b^2) \ - \ \theta_\ell[(b+d)^2 + (a+c)^2], \end{aligned}$$

where the angles are

$$\theta_h = \tan^{-1}\left(\frac{2cb}{d^2 - c^2}\right), \quad \theta_\ell = \tan^{-1}\left(\frac{2(b+d)(a+c)}{(b+d)^2 - (a+c)^2}\right),$$

$$\theta_t = \tan^{-1}\left(\frac{2ab}{b^2 - a^2}\right), \quad \theta_r = \tan^{-1}\left(\frac{2(b-d)(a-c)}{(b-d)^2 - (a-c)^2}\right).$$

26. A method as recited in claim 22 wherein the intermediate transform is a wavelet transform based on arithmetic wavelets, and the calculation of the intermediate transform includes the step of renormalization.

27. A method as recited in claim 22 wherein the primary transformation consists of the Fourier transform, and the intermediate transform is a wavelet transformation based on modular wavelets.

28. A method as recited in claim 22 wherein the intermediate transform is a wavelet transform based on modular wavelets, and the calculation of the intermediate transform includes the step of renormalization.

29. A method as recited in claim 22 wherein the primary transformation consists of the Fourier transform, and the intermediate transform is a wavelet transformation based on fan wavelets.

30. A method as recited in claim 22 wherein the intermediate transform is a wavelet transform based on fan wavelets, and the calculation of the intermediate transform includes the step of renormalization.

31. A method as recited in claim 22 wherein the sampling of data at the points of the Farey quadrature includes temporary storage of sequences of data in a buffer.

32. A method as recited in claim 22 wherein the input data is multi-dimensional.

33. A method as recited in claim 22 wherein the constants and coefficients required in the calculation of the intermediate transform and in the calculation of the primary transform are archived and recovered from memory at run time.

34. A method of data processing comprising the steps of: providing input data in an initial form; sampling the input data at the points of the Farey quadrature; transforming the Farey-sampled data to a second form; and processing the data in the second form.

35. A method as recited in claim 34 wherein: the step of transforming the Farey-sampled data into the second form is achieved via a wavelet transformation; and the second form of the Farey-sampled data takes the form of wavelet coefficients.

36. A method as recited in claim 35 wherein the wavelet transformation is based on arithmetic wavelets.

37. A method as recited in claim 36 wherein the arithmetic wavelets are defined in accordance with the following expressions:

$$\tilde{\vartheta}_I(\theta) = \begin{cases} +2\cos\theta + 2\sin\theta - 2; & \text{if } 0 \le \theta \le \frac{\pi}{2}, \\ +2\cos\theta - 2\sin\theta + 2; & \text{if } \frac{\pi}{2} \le \theta \le \pi, \\ -2\cos\theta - 2\sin\theta - 2; & \text{if } \pi \le \theta \le \frac{3\pi}{2}, \\ -2\cos\theta + 2\sin\theta + 2; & \text{if } \frac{3\pi}{2} \le \theta \le 2\pi; \end{cases}$$

and if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ labels an arithmetic arrow;

and if A labels a top arithmetic arrow and $a > c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(d^2-c^2)\cos\theta + 8cd\sin\theta - 4(d^2+c^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +2(a^2+d^2-b^2-c^2+2ac-2bd)\cos\theta \\ \quad +4(cd-ab-bc-ad)\sin\theta \\ \quad +2(a^2+b^2-c^2-d^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+d^2-a^2-c^2+2ac-2bd)\cos\theta \\ \quad +4(ab+cd-ad-bc)\sin\theta \\ \quad +2(-a^2-b^2-c^2-d^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ 0; \qquad \text{otherwise}; \end{cases}$$

and if A labels a top arithmetic arrow and $c \geq a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2-2ac+2bd)\cos\theta \\ \quad +4(ad+bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(a^2+d^2-b^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd-ab)\sin\theta \\ \quad +2(a^2+b^2-c^2-d^2-2bd-2ac); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +4(a^2-b^2)\cos\theta - 8ab\sin\theta + 4(a^2+b^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ 0; \qquad \text{otherwise}; \end{cases}$$

and if A labels a bottom arithmetic arrow and $a > -c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(-ad-bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+c^2-a^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(ab-ad-bc-cd)\sin\theta \\ \quad +\,2(c^2+d^2-a^2-b^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +4\,(c^2-d^2)\cos\theta \;-\; 8cd\sin\theta \;+\; 4(c^2+d^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ 0; \qquad\qquad \text{otherwise}; \end{cases}$$

and if A labels a bottom arithmetic arrow and $-c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(b^2-a^2)\cos\theta \;+\; 8ab\sin\theta \;-\; 4(a^2+b^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +2(b^2+c^2-a^2-d^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+ab-cd)\sin\theta \\ \quad +\,2(c^2+d^2-a^2-b^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(b^2+d^2-a^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd+ab)\sin\theta \\ \quad +\,2(-a^2-b^2-c^2-d^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ 0; \qquad\qquad \text{otherwise.} \end{cases}$$

38. A method as recited in claim 35 wherein the wavelet transformation is based on modular wavelets.

39. A method as recited in claim 38 wherein the modular wavelets are defined in accordance with the following expressions:

$$\psi_A(\theta) = \sum_{n\ge 0} n\,\bar{\vartheta}_{U^n A}(\theta),$$
$$\psi_{SA}(\theta) = \sum_{n\ge 0} n\,\bar{\vartheta}_{T^{-n} A}(\theta),$$

where

$$\bar{\vartheta}_A(\theta) = \begin{cases} \tilde{\vartheta}_A(\theta); & \text{if } A \ne U^{-1}, T^{-1}, \\ \tilde{\vartheta}_{U^{-1}}(\theta) - 2\sin\theta; & \text{if } A = U^{-1}, \\ \tilde{\vartheta}_{T^{-1}}(\theta) + 2\sin\theta; & \text{if } A = T^{-1}, \end{cases}$$

with $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, - $= \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$; and

$$\tilde{\vartheta}_I(\theta) = \begin{cases} +2\cos\theta + 2\sin\theta - 2; & \text{if } 0 \le \theta \le \frac{\pi}{2}, \\ +2\cos\theta - 2\sin\theta + 2; & \text{if } \frac{\pi}{2} \le \theta \le \pi, \\ -2\cos\theta - 2\sin\theta - 2; & \text{if } \pi \le \theta \le \frac{3\pi}{2}, \\ -2\cos\theta + 2\sin\theta + 2; & \text{if } \frac{3\pi}{2} \le \theta \le 2\pi; \end{cases}$$

and if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ labels an arithmetric arrow;

and if A labels a top arithmetic arrow and $a > c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(d^2 - c^2)\cos\theta + 8cd\sin\theta - 4(d^2 + c^2); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +2(a^2 + d^2 - b^2 - c^2 + 2ac - 2bd)\cos\theta \\ \quad +4(cd - ab - bc - ad)\sin\theta \\ \quad + 2(a^2 + b^2 - c^2 - d^2 + 2ac + 2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2 + d^2 - a^2 - c^2 + 2ac - 2bd)\cos\theta \\ \quad +4(ab + cd - ad - bc)\sin\theta \\ \quad + 2(-a^2 - b^2 - c^2 - d^2 + 2ac + 2bd); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ 0; \qquad \text{otherwise;} \end{cases}$$

and if A labels a top arithmetic arrow and $c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2 + c^2 - b^2 - d^2 - 2ac + 2bd)\cos\theta \\ \quad +4(ad + bc - ab - cd)\sin\theta \\ \quad +2(a^2 + b^2 + c^2 + d^2 - 2ac - 2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(a^2 + d^2 - b^2 - c^2 + 2bd - 2ac)\cos\theta \\ \quad +4(ad + bc + cd - ab)\sin\theta \\ \quad + 2(a^2 + b^2 - c^2 - d^2 - 2bd - 2ac); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +4(a^2 - b^2)\cos\theta - 8ab\sin\theta + 4(a^2 + b^2); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ 0; \qquad \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $a > -c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(-ad-bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+c^2-a^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(ab-ad-bc-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +4(c^2-d^2)\cos\theta - 8cd\sin\theta + 4(c^2+d^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ 0; \qquad\qquad \text{otherwise}; \end{cases}$$

and if A labels a bottom arithmetic arrow and $-c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(b^2-a^2)\cos\theta + 8ab\sin\theta - 4(a^2+b^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +2(b^2+c^2-a^2-d^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+ab-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(b^2+d^2-a^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd+ab)\sin\theta \\ \quad +2(-a^2-b^2-c^2-d^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ 0; \qquad\qquad \text{otherwise}. \end{cases}$$

40. A method as recited in claim 35 wherein the wavelet transformation is based on fan wavelets.

41. A method as recited in claim 40 wherein the fan wavelets are defined in accordance with the following expressions:

$$\phi_A(\theta) = \sum_{n\ge 0} \overline{\vartheta}_{U^n A}(\theta),$$
$$\phi_{SA}(\theta) = \sum_{n\ge 0} \overline{\vartheta}_{T^{-n} A}(\theta),$$

where

$$\overline{\vartheta}_A(\theta) = \begin{cases} \tilde{\vartheta}_A(\theta); & \text{if } A \ne U^{-1}, T^{-1}, \\ \tilde{\vartheta}_{U^{-1}}(\theta) - 2\sin\theta; & \text{if } A = U^{-1}, \\ \tilde{\vartheta}_{T^{-1}}(\theta) + 2\sin\theta; & \text{if } A = T^{-1}, \end{cases}$$

with $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$; and

$$\tilde{\vartheta}_I(\theta) = \begin{cases} +2\cos\theta + 2\sin\theta - 2; & \text{if } 0 \le \theta \le \frac{\pi}{2}, \\ +2\cos\theta - 2\sin\theta + 2; & \text{if } \frac{\pi}{2} \le \theta \le \pi, \\ -2\cos\theta - 2\sin\theta - 2; & \text{if } \pi \le \theta \le \frac{3\pi}{2}, \\ -2\cos\theta + 2\sin\theta + 2; & \text{if } \frac{3\pi}{2} \le \theta \le 2\pi; \end{cases}$$

and if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ labels an arithmetic arrow;

and if A labels a top arithmetic arrow and $a > c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(d^2 - c^2)\cos\theta + 8cd\sin\theta - 4(d^2 + c^2); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +2(a^2 + d^2 - b^2 - c^2 + 2ac - 2bd)\cos\theta \\ \quad +4(cd - ab - bc - ad)\sin\theta \\ \quad + 2(a^2 + b^2 - c^2 - d^2 + 2ac + 2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2 + d^2 - a^2 - c^2 + 2ac - 2bd)\cos\theta \\ \quad +4(ab + cd - ad - bc)\sin\theta \\ \quad + 2(-a^2 - b^2 - c^2 - d^2 + 2ac + 2bd); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ 0; \qquad \text{otherwise}; \end{cases}$$

and if A labels a top arithmetic arrow and $c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2 + c^2 - b^2 - d^2 - 2ac + 2bd)\cos\theta \\ \quad +4(ad + bc - ab - cd)\sin\theta \\ \quad +2(a^2 + b^2 + c^2 + d^2 - 2ac - 2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(a^2 + d^2 - b^2 - c^2 + 2bd - 2ac)\cos\theta \\ \quad +4(ad + bc + cd - ab)\sin\theta \\ \quad + 2(a^2 + b^2 - c^2 - d^2 - 2bd - 2ac); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +4(a^2 - b^2)\cos\theta - 8ab\sin\theta + 4(a^2 + b^2); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ 0; \qquad \text{otherwise}; \end{cases}$$

and if A labels a bottom arithmetic arrow and $a > -c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(-ad-bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+c^2-a^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(ab-ad-bc-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +4\,(c^2-d^2)\cos\theta \;-\; 8cd\sin\theta \;+\; 4(c^2+d^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ 0; \qquad\qquad \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $-c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(b^2-a^2)\cos\theta \;+\; 8ab\sin\theta \;-\; 4(a^2+b^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +2(b^2+c^2-a^2-d^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+ab-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(b^2+d^2-a^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd+ab)\sin\theta \\ \quad +2(-a^2-b^2-c^2-d^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ 0; \qquad\qquad \text{otherwise.} \end{cases}$$

42. A method as recited in claim 35 wherein the steps of sampling the input data at the points of the Farey quadrature and transforming the Farey-sampled into wavelet coefficients is characterized by a binary cascade of arrow structures arising from the calculation of wavelet coefficients by sampling the input data in its initial form at the points of the Farey quadrature as defined along a circle in the complex plane.

43. A method as recited in claim 42 wherein the wavelet transformation is based on arithmetic wavelets and the step of transforming the Farey-sampled data into arithmetic wavelet coefficients includes the step of regularization.

44. A method as recited in claim 35 wherein the wavelet transformation is based on a finite number of wavelets.

45. A method as recited in claim 35 wherein: wavelet transformation is achieved via a plurality of wavelet transformation operations each for a specified arc along a circle in

the complex plane; anc e plurality of wavelet transformation ope ons are calculated in parallel with each other.

46. A method as recited in claim 34 wherein the input data is digital data.

47. A method as recited in claim 34 wherein the input data is analog date and the analog input data is sampled by placing analog sensors at locations corresponding to the points of the Farey quadrature.

48. A method as recited in claim 34 wherein the input data is multi-dimensional.

49. A method as recited in claim 34 wherein the sampling of data at the points of the Farey quadrature includes temporary storage of sequences of data in a buffer.

50. A digital processing system comprising a digital filter that receives an input signal and outputs a transformed signal, the digital filter being an arithmetic wavelet filter in which the arithmetic wavelets are defined in accordance with the following expressions:

$$\tilde{\vartheta}_I(\theta) = \begin{cases} +2\cos\theta + 2\sin\theta - 2; & \text{if } 0 \le \theta \le \frac{\pi}{2}, \\ +2\cos\theta - 2\sin\theta + 2; & \text{if } \frac{\pi}{2} \le \theta \le \pi, \\ -2\cos\theta - 2\sin\theta - 2; & \text{if } \pi \le \theta \le \frac{3\pi}{2}, \\ -2\cos\theta + 2\sin\theta + 2; & \text{if } \frac{3\pi}{2} \le \theta \le 2\pi; \end{cases}$$

and if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ labels an arithmetic arrow;

and if A labels a top arithmetic arrow and $a > c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(d^2-c^2)\cos\theta + 8cd\sin\theta - 4(d^2+c^2); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +2(a^2+d^2-b^2-c^2+2ac-2bd)\cos\theta \\ \quad +4(cd-ab-bc-ad)\sin\theta \\ \quad +2(a^2+b^2-c^2-d^2+2ac+2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+d^2-a^2-c^2+2ac-2bd)\cos\theta \\ \quad +4(ab+cd-ad-bc)\sin\theta \\ \quad +2(-a^2-b^2-c^2-d^2+2ac+2bd); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ 0; \qquad \text{otherwise;} \end{cases}$$

and if A labels a top arithmetic arrow and $c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2-2ac+2bd)\cos\theta \\ \quad +4(ad+bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2-2ac-2bd); & \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(a^2+d^2-b^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd-ab)\sin\theta \\ \quad +2(a^2+b^2-c^2-d^2-2bd-2ac); & \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +4(a^2-b^2)\cos\theta - 8ab\sin\theta + 4(a^2+b^2); & \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ 0; & \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $a > -c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(-ad-bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2+2ac+2bd); & \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+c^2-a^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(ab-ad-bc-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2+2ac+2bd); & \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +4(c^2-d^2)\cos\theta - 8cd\sin\theta + 4(c^2+d^2); & \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ 0; & \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $-c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(b^2-a^2)\cos\theta + 8ab\sin\theta - 4(a^2+b^2); & \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +2(b^2+c^2-a^2-d^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+ab-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2-2ac-2bd); & \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(b^2+d^2-a^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd+ab)\sin\theta \\ \quad +2(-a^2-b^2-c^2-d^2-2ac-2bd); & \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ 0; & \text{otherwise.} \end{cases}$$

51. A digital processing system comprising a digital filter that receives an input signal and outputs a transformed signal, the digital filter being a modular wavelet filter in which the modular wavelets are defined in accordance with the following expressions:

$$\psi_A(\theta) = \sum_{n\geq 0} n\ \overline{\vartheta}_{U^n A}(\theta),$$
$$\psi_{SA}(\theta) = \sum_{n\geq 0} n\ \overline{\vartheta}_{T^{-n} A}(\theta),$$

where

$$\overline{\vartheta}_A(\theta) = \begin{cases} \tilde{\vartheta}_A(\theta); & \text{if } A \neq U^{-1}, T^{-1}, \\ \tilde{\vartheta}_{U^{-1}}(\theta) - 2\sin\theta; & \text{if } A = U^{-1}, \\ \tilde{\vartheta}_{T^{-1}}(\theta) + 2\sin\theta; & \text{if } A = T^{-1}, \end{cases}$$

with $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $U = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$; and

$$\tilde{\vartheta}_I(\theta) = \begin{cases} +2\cos\theta + 2\sin\theta - 2; & \text{if } 0 \leq \theta \leq \frac{\pi}{2}, \\ +2\cos\theta - 2\sin\theta + 2; & \text{if } \frac{\pi}{2} \leq \theta \leq \pi, \\ -2\cos\theta - 2\sin\theta - 2; & \text{if } \pi \leq \theta \leq \frac{3\pi}{2}, \\ -2\cos\theta + 2\sin\theta + 2; & \text{if } \frac{3\pi}{2} \leq \theta \leq 2\pi; \end{cases}$$

and if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ labels an arithmetic arrow;

and if A labels a top arithmetic arrow and $a > c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(d^2 - c^2)\cos\theta + 8cd\sin\theta - 4(d^2 + c^2); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2 - c^2} \leq \theta \leq \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2 - (a+c)^2}, \\ +2(a^2 + d^2 - b^2 - c^2 + 2ac - 2bd)\cos\theta \\ \quad +4(cd - ab - bc - ad)\sin\theta \\ \quad +2(a^2 + b^2 - c^2 - d^2 + 2ac + 2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2 - (a+c)^2} \leq \theta \leq \tan^{-1}\frac{2ba}{b^2 - a^2}, \\ +2(b^2 + d^2 - a^2 - c^2 + 2ac - 2bd)\cos\theta \\ \quad +4(ab + cd - ad - bc)\sin\theta \\ \quad +2(-a^2 - b^2 - c^2 - d^2 + 2ac + 2bd); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2 - a^2} \leq \theta \leq \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2 - (c-a)^2}, \\ 0; \qquad \text{otherwise}; \end{cases}$$

and if A labels a top arithmetic arrow and $c \geq a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2-2ac+2bd)\cos\theta \\ \quad +4(ad+bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2-2ac-2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ \\ +2(a^2+d^2-b^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd-ab)\sin\theta \\ \quad +2(a^2+b^2-c^2-d^2-2bd-2ac); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ \\ +4(a^2-b^2)\cos\theta \; - \; 8ab\sin\theta \; + \; 4(a^2+b^2); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ \\ 0; \qquad \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $a > -c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(-ad-bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2+2ac+2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ \\ +2(b^2+c^2-a^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(ab-ad-bc-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2+2ac+2bd); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ \\ +4\,(c^2-d^2)\cos\theta \; - \; 8cd\sin\theta \; + \; 4(c^2+d^2); \\ \qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ \\ 0; \qquad \text{otherwise;} \end{cases}$$

and if A labels a bottom arithmetic arrow and $-c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(b^2-a^2)\cos\theta \; + \; 8ab\sin\theta \; - \; 4(a^2+b^2); \\ \qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ \\ +2(b^2+c^2-a^2-d^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+ab-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2-2ac-2bd); \\ \qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ \\ +2(b^2+d^2-a^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd+ab)\sin\theta \\ \quad +2(-a^2-b^2-c^2-d^2-2ac-2bd); \\ \qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ \\ 0; \qquad \text{otherwise.} \end{cases}$$

52. A digital processin ystem comprising a digital filter that rec s an input signal and outputs a transformed signal, the digital filter being a fan wavelet filter in which the fan wavelets are defined in accordance with the following expressions:

$$\phi_A(\theta) = \sum_{n \geq 0} \overline{\vartheta}_{U^n A}(\theta),$$
$$\phi_{SA}(\theta) = \sum_{n \geq 0} \overline{\vartheta}_{T^{-n} A}(\theta),$$

where

$$\overline{\vartheta}_A(\theta) = \begin{cases} \tilde{\vartheta}_A(\theta); & \text{if } A \neq U^{-1}, T^{-1}, \\ \tilde{\vartheta}_{U^{-1}}(\theta) - 2\sin\theta; & \text{if } A = U^{-1}, \\ \tilde{\vartheta}_{T^{-1}}(\theta) + 2\sin\theta; & \text{if } A = T^{-1}, \end{cases}$$

with $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $U = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$; and

$$\tilde{\vartheta}_I(\theta) = \begin{cases} +2\cos\theta + 2\sin\theta - 2; & \text{if } 0 \leq \theta \leq \frac{\pi}{2}, \\ +2\cos\theta - 2\sin\theta + 2; & \text{if } \frac{\pi}{2} \leq \theta \leq \pi, \\ -2\cos\theta - 2\sin\theta - 2; & \text{if } \pi \leq \theta \leq \frac{3\pi}{2}, \\ -2\cos\theta + 2\sin\theta + 2; & \text{if } \frac{3\pi}{2} \leq \theta \leq 2\pi; \end{cases}$$

and if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ labels an arithmetic arrow;

and if A labels a top arithmetic arrow and $a > c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(d^2 - c^2)\cos\theta + 8cd\sin\theta - 4(d^2 + c^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2cd}{d^2 - c^2} \leq \theta \leq \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2 - (a+c)^2}, \\ +2(a^2 + d^2 - b^2 - c^2 + 2ac - 2bd)\cos\theta \\ \quad +4(cd - ab - bc - ad)\sin\theta \\ \quad + 2(a^2 + b^2 - c^2 - d^2 + 2ac + 2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2 - (a+c)^2} \leq \theta \leq \tan^{-1}\frac{2ba}{b^2 - a^2}, \\ +2(b^2 + d^2 - a^2 - c^2 + 2ac - 2bd)\cos\theta \\ \quad +4(ab + cd - ad - bc)\sin\theta \\ \quad + 2(-a^2 - b^2 - c^2 - d^2 + 2ac + 2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2 - a^2} \leq \theta \leq \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2 - (c-a)^2}, \\ 0; \qquad\qquad \text{otherwise}; \end{cases}$$

and if A labels a top arithmetic arrow and $c \geq a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2-2ac+2bd)\cos\theta \\ \quad +4(ad+bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(a^2+d^2-b^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd-ab)\sin\theta \\ \quad +2(a^2+b^2-c^2-d^2-2bd-2ac); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ +4(a^2-b^2)\cos\theta \;-\; 8ab\sin\theta \;+\; 4(a^2+b^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ 0; \qquad\qquad \text{otherwise}; \end{cases}$$

and if A labels a bottom arithmetic arrow and $a > -c$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +2(a^2+c^2-b^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(-ad-bc-ab-cd)\sin\theta \\ \quad +2(a^2+b^2+c^2+d^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2} \le \theta \le \tan^{-1}\frac{2ba}{b^2-a^2}, \\ +2(b^2+c^2-a^2-d^2+2ac-2bd)\cos\theta \\ \quad +4(ab-ad-bc-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2+2ac+2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +4\,(c^2-d^2)\cos\theta \;-\; 8cd\sin\theta \;+\; 4(c^2+d^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ 0; \qquad\qquad \text{otherwise}; \end{cases}$$

and if A labels a bottom arithmetic arrow and $-c \ge a$, then

$$\tilde{\vartheta}_A(\theta) = \begin{cases} +4(b^2-a^2)\cos\theta \;+\; 8ab\sin\theta \;-\; 4(a^2+b^2); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2ba}{b^2-a^2} \le \theta \le \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2}, \\ +2(b^2+c^2-a^2-d^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+ab-cd)\sin\theta \\ \quad +2(c^2+d^2-a^2-b^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2(d-b)(c-a)}{(d-b)^2-(c-a)^2} \le \theta \le \tan^{-1}\frac{2cd}{d^2-c^2}, \\ +2(b^2+d^2-a^2-c^2+2bd-2ac)\cos\theta \\ \quad +4(ad+bc+cd+ab)\sin\theta \\ \quad +2(-a^2-b^2-c^2-d^2-2ac-2bd); \\ \qquad\qquad \text{if } \tan^{-1}\frac{2cd}{d^2-c^2} \le \theta \le \tan^{-1}\frac{2(b+d)(a+c)}{(b+d)^2-(a+c)^2}, \\ 0; \qquad\qquad \text{otherwise}. \end{cases}$$

53. A method of obtai .g an inverse transform of input digital a comprising the steps of: providing input digital data in an initial form; transforming the data in the initial form to an intermediate form; and transforming the intermediate form of the data using an intermediate inverse transform to obtain output values at the points of the Farey quadrature.

54. A method as recited in claim 53 wherein the input digital data is in the form of Fourier coefficients and the combination of transforming the data from the Fourier coefficients to the intermediate form and from the intermediate form to obtain output values at the points of the Farey quadrature constitutes an inverse Fourier transform.

55. A method as recited in claim 54 wherein the intermediate form of the data is in the form of wavelet coefficients.

56. A method as recited in claim 55 wherein the wavelet coefficients are coefficients for arithmetic wavelets,

$$\tilde{\vartheta}_A(\theta) = \begin{cases} \overline{\vartheta}_A(\theta); & \text{if } A \neq U^{-1}, T^{-1}, \\ \overline{\vartheta}_{U^{-1}}(\theta) + 2\sin\theta; & \text{if } A = U^{-1}, \\ \overline{\vartheta}_{T^{-1}}(\theta) - 2\sin\theta; & \text{if } A = T^{-1}, \end{cases}$$

with $U^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, and $T^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$, and conversion of the data from the initial form to the intermediate form is based on the following expressions:

$$\begin{aligned} e^{in\theta} &= \cos n\theta + i \sin n\theta \\ &= -\left[c_0^n + c_1^n e^{i\theta} + c_{-1}^n e^{-i\theta}\right] + \frac{i}{4} \sum_A \left\{ n(\xi^n + \eta^n) + \frac{\eta + \xi}{\eta - \xi} (\xi^n - \eta^n) \right\} \overline{\vartheta}_A(\theta), \end{aligned}$$

where the sum is over all arithmetic arrows, the underlying chord of which has complex endpoints ξ, η, and

$$c_0^n = \begin{cases} -1, & n \equiv 0(4); \\ 0, & n \equiv 1(4); \\ -1, & n \equiv 2(4); \\ 0, & n \equiv 3(4); \end{cases} \quad c_1^n = \begin{cases} 0, & n \equiv 0(4); \\ -1, & n \equiv 1(4); \\ i, & n \equiv 2(4); \\ 0, & n \equiv 3(4); \end{cases} \quad c_{-1}^n = \begin{cases} 0, & n \equiv 0(4); \\ 0, & n \equiv 1(4); \\ -i, & n \equiv 2(4); \\ -1, & n \equiv 3(4). \end{cases}$$

57. A method as recited in claim 55 wherein the wavelet coefficients are coefficients for modular wavelets, ψ_A, and conversion of the data from the initial form to the intermediate form is based on the following expressions:

$$\begin{aligned} e^{in\theta} &= \cos n\theta + i \sin n\theta \\ &= -\left[c_0^n + c_1^n e^{i\theta} + c_{-1}^n e^{-i\theta}\right] \\ &\quad + \frac{i}{4} \sum_A \left\{ n(\xi^n + \eta^n) + \frac{\eta + \xi}{\eta - \xi} (\xi^n - \eta^n) \right\} \left[\psi_{UA}(\theta) - 2\psi_A(\theta) + \psi_{U^{-1}A}(\theta)\right], \end{aligned}$$

where the sum is over all arithmetic arrows, the underlying chord of which has complex endpoints ξ, η, where $U^{\pm 1} = \begin{pmatrix} 1 & 0 \\ \pm 1 & 1 \end{pmatrix}$, and where

$$c_0^n = \begin{cases} -1, & n \equiv 0(4); \\ 0, & n \equiv 1(4); \\ -1, & n \equiv 2(4); \\ 0, & n \equiv 3(4); \end{cases} \quad c_1^n = \begin{cases} 0, & n \equiv 0(4); \\ -1, & n \equiv 1(4); \\ i, & n \equiv 2(4); \\ 0, & n \equiv 3(4); \end{cases} \quad c_{-1}^n = \begin{cases} 0, & n \equiv 0(4); \\ 0, & n \equiv 1(4); \\ -i, & n \equiv 2(4); \\ -1, & n \equiv 3(4). \end{cases}$$

58. A method as recited in claim 55 wherein the wavelet coefficients are coefficients for fan wavelets, ϕ_A, and conversion of the data from the initial form to the intermediate form is based on the following expressions:

$$\begin{aligned} e^{in\theta} &= \cos n\theta + i \sin n\theta \\ &= -\left[c_0^n + c_1^n e^{i\theta} + c_{-1}^n e^{-i\theta}\right] \\ &\quad + \frac{i}{4} \sum_A \left\{ n(\xi^n + \eta^n) + \frac{\eta + \xi}{\eta - \xi} (\xi^n - \eta^n) \right\} \left[\phi_A(\theta) - \phi_{UA}(\theta)\right], \end{aligned}$$

where the sum is over all arithmetic arrows, the underlying chord of which has complex endpoints ξ, η; where $U = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, and where

$$c_0^n = \begin{cases} -1, & n \equiv 0(4); \\ 0, & n \equiv 1(4); \\ -1, & n \equiv 2(4); \\ 0, & n \equiv 3(4); \end{cases} \quad c_1^n = \begin{cases} 0, & n \equiv 0(4); \\ -1, & n \equiv 1(4); \\ i, & n \equiv 2(4); \\ 0, & n \equiv 3(4); \end{cases} \quad c_{-1}^n = \begin{cases} 0, & n \equiv 0(4); \\ 0, & n \equiv 1(4); \\ -i, & n \equiv 2(4); \\ -1, & n \equiv 3(4). \end{cases}$$

59. A method as recited in claim 53 wherein the intermediate form of the data is in the form of wavelet coefficients.

60. A method as recited in claim 53 wherein the initial form of the input digital data is Fourier coefficients, the intermediate form of the data is in the form of wavelet coefficients, and the output values at the points of the Farey quadrature are obtained via the use of a binary cascade of arrow structures arising from the calculation of wavelet coefficients as defined along a circle in the complex plane.

61. A method as recited in claim 60 further comprising the step of storing terminal arrow structures in order for restart or iterative refinement.

62. A method as recited in claim 59 wherein the wavelet coefficients are coefficients for arithmetic wavelets.

63. A method as recited in claim 59 wherein the wavelet coefficients are coefficients for modular wavelets.

64. A method as recited in claim 59 wherein the wavelet coefficients are coefficients for fan wavelets.

65. A method as recited in claim 59 wherein the input data is multi-dimensional.